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EXPERTS PAPER

ESG ACCELERATION

# *ESG: ACCELERATION OF THE PRODUCTION PROCESS IN MULTI REGULATORY STANDARDS FRAMEWORK*

Regulatory standards require insurers to evaluate the economic value of their balance sheet in a market consistent way. To that end, insurers use Economic Scenario Generators (ESG) to assess different economic risk factors and project financial returns cashflows.

Recent ESGs represent each risk factor using increasingly complex financial models which induce costly challenges in the production process. This complexity is amplified in a multi-norm framework where insurers are compelled to generate intensely multiple economic scenarios tables using different assumptions.

This article defines an integrated production process to accelerate ESG tables production in multi standard framework. It identifies conceivable acceleration methodologies accordingly to the sensitivities to be produced. It finally discusses the operational implementation of the acceleration process.

## 1. ESG: THE COMPLEXITY BEHIND A NECESSARY TOOL

### 1. 1. Standard ESG production process

Under Solvency II or IFRS 17 regulatory standards insurers apply market-consistent approaches to evaluate their balance sheets.

The use of ESGs and Monte-Carlo methods is thus necessary to evaluate the cost of financial options and guarantees embedded in life insurance policies.

Several economic risk factors can be modelled in an ESG: rates, equities, real-estate, inflation and credit spreads, using stochastic financial models and following a specific production process: calibration, diffusion and validation.

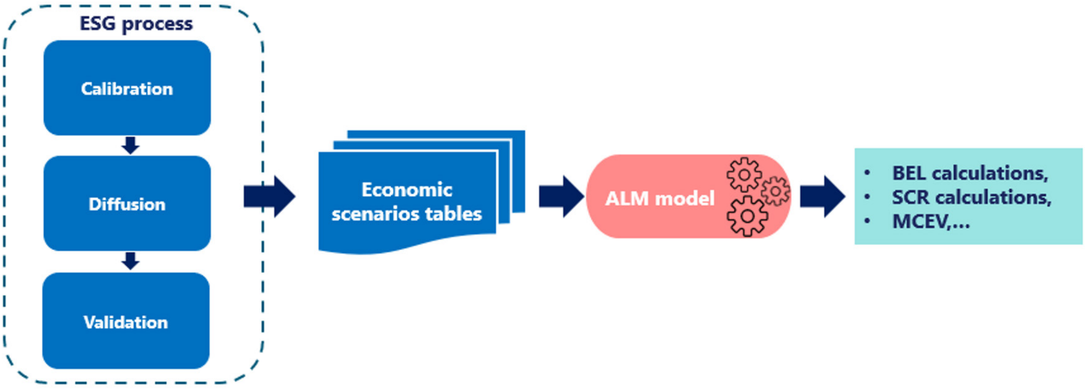



*Figure 1 – Use of ESG in ALM process*

The first step requires market consistent fitting of stochastic model's parameters to spot market data when listed or to historical market data (e.g. calibration of real estate models). With the use of correlation matrix, calibrated parameters are then injected in stochastic differential equations to project multiple scenarios of financial returns. Finally, generated scenarios undergo many statistical and Monte-Carlo tests to insure their martingality and market consistency.

## 1.2. ESG complexity

Increasingly complex financial models are used in recent ESGs to satisfy regulatory requirements and generate scenarios consistent with the changing market context. These models engender many challenges to insurers at each step of the production process, and especially at calibration.

In fact, calibration of highly parametrized models is complex as the optimization algorithms estimate at each iteration theoretical semi-closed formulas. The optimization problem become extremely complicated, and its convergence is time consuming. Furthermore, calibrated parameters are often saturated, i.e. occurrence of inaccurate calibrations where parameters are trapped in interval bounds defined in the optimization algorithm, leading to market consistency problems.

## 1.3. ESG operational challenges in multi-standard context

Along with the technical challenges of the complex models used in the ESG, the intensity of production increases drastically in a multi-standard framework, where insurers generate several economic scenarios tables for their quarterly or annual statements production or for their own studies. These sensitivities include either a movement of the yield curve or the volatility levels, for instance:

- **Solvency II** closings: ESG tables using rate curves with and without volatility adjustment (VA) and applying standard formula chocs.
- **IFRS 17** closing: ESG tables using rate curves with different VA levels according to the portfolio and for AoC.
- **ESG** tables for ORSA sensitivities
- **MCEV** calculations: ESG tables with different interest rates and volatility chocs

The standard production process becomes operationally intensive and can be accelerated by directly adjusting the economic scenario tables from a central reference table parameters or scenarios. Furthermore, ESG calibrated parameters and scenarios produced for annual statements could be adjusted to accelerate quarterly closings.

# 2. ESG: ACCELERATION PRODUCTION PROCESS

Many acceleration methodologies are conceivable accordingly to the sensitivities to produce:

- Forward adjustments of a reference table, used for nominal rates yield movements.
- Adjustment of a reference table parameters without recalibration, used for sensitivities impacting volatility assumptions.
- Adjustment of equity trajectories without recalibration or regeneration, used for sensitivities affecting equity volatility assumptions.
- Pre-processing of calibration on a very large number of initial conditions and construction of a calibration reference frame to select the "nearest neighbour" parameters or to calibrate a proxy base on Machine Learning

This article displays the two first approaches.

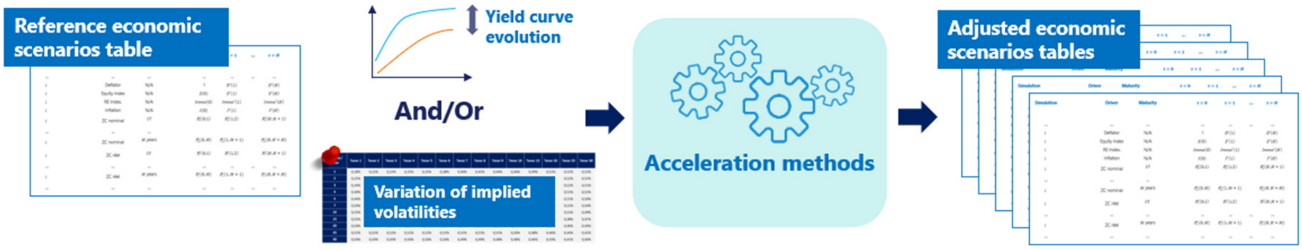



*Figure 2 - Acceleration ESG production process*

## 2.1. Forward rate adjustments method

This methodology consists of adjusting a reference table following a change in the level of initial rates while implied volatilities levels remain the same. It could be used to produce tables for solvency 2 closings (interest rate shocks and with/without VA sensitivities), IFRS 17 VA sensitivities, ORSA sensitivities.
The approach applies forward zero-coupon prices ratios to the reference table and produce new economic scenarios without recalibration or regeneration.
The forward zero-coupon price ratio includes ante/poste evolution of the yield curve:

$$\frac{\frac{P_n^*(0,t+m)}{P_n^*(0,t)}}{\frac{P_n(0,t+m)}{P_n(0,t)}}$$

For instance, adjusted deflator or ZC nominal prices scenarios are expressed below:

**Deflator:**

$$D^{i,*}(t) = D^{i}(t) \times \frac{P_n^*(0,t)}{P_n(0,t)}$$

**Nominal ZC price:**

$$P_n^{i,*}(t,t+m) = P_n^{i}(t,t+m) \times \frac{\frac{P_n^*(0,t+m)}{P_n(0,t+m)}}{\frac{P_n^*(0,t)}{P_n(0,t)}}$$

Where $P_n^i(t,t+m)$ (resp. $P_n^{i,*}(t,t+m)$ ): the $i^{th}$ ZC price scenario of maturity $\mathrm{m}$ at instant $\mathrm{t}$ generated by the reference table (resp. the adjusted table).
Formulas following the same principle can be applied to the other risk variables: equity, real-estate, inflation, real ZC price and risked rates.

| Maturity/Tenor | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,01% | 0,01% | 0,01% | 0,00% |
| 2 | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,01% | 0,01% | 0,00% |
| 3 | 0,00% | 0,01% | 0,01% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% |
| 4 | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% |
| 5 | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 0,01% |
| 7 | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,00% | 0,00% | 0,01% | 0,01% |
| 10 | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% |
| 15 | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,02% |
| 20 | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,02% |
| 25 | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,02% | 0,02% |
| 30 | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,01% | 0,02% | 0,02% |

*Figure 3 - absolute differences between implied Monte-Carlo normal swaption volatilities ante/post +200 Bps evolution of the yield curve*

The adjusted scenarios remain perfectly martingale by construction in this approach and their market consistency is not altered. In practice, ante/post implied Monte-Carlo normal swaptions volatilities vary slightly even for a considerable movement of rate levels as shown in the matrix above.

## 2.2. Model parameters adjustment method

This approach consists of adjusting directly ESG model parameters following to a change in volatility level assumptions. It is suitable for sensitivities with volatility shocks or for market data changes between two closings. However, scenario tables need to be regenerated with the new adjusted parameters.

Without loss of generality, the model parameters adjustment approach is presented for the LMM+ in the ensuing. The LMM+ and its formulas are introduced in the Annex.

Noting $S_{\alpha,\beta}(t)$ (resp. $\sigma_{\alpha,\beta}$) the swap forward rate (resp. the normal swaption volatility) at the instant t maturing a $T_\alpha$ with a tenor $T_\beta - T_\alpha$, the following approximation can be found below:

$$dS_{\alpha,\beta}(t) \approx \sum_{k=\alpha}^{\beta-1} w_k^{\alpha,\beta}(0)(F_k(t)+\delta)\ .\sum_{q=1}^{2} \xi_k^q(t)dZ_q^{\alpha,\beta}(t)$$

Where $w_k^{\alpha,\beta}$ is a weight function.

It is possible to deduct a proportional choc factor between the new assumptions of swaption volatilities$(\sigma^*_{\alpha,\beta})$ and the reference swaption volatilities$(\sigma_{\alpha,\beta})$ :

$$\Delta_{vol} = \frac{\sum \sigma^*_{i,j}}{\sum \sigma_{i,j}}$$

LMM+ parameters can be adjusted following either the two approaches below:

1. Adjustment of Rebonato function parameters:
   $$g_j^*(t) = \left(a \times \Delta_{vol} + b \times \Delta_{vol}\,.T_{j-m(t)}\right)e^{-cT_{j-m(t)}} + d \times \Delta_{vol}$$
2. Adjustment of stochastic Variance CIR process parameters:

$$V_0{}^* = \Delta_{vol}{}^2 \times V_0 \quad ; \quad \theta^* = \Delta_{vol}{}^2 \times \theta \quad ; \quad \epsilon^* = \Delta_{vol} \times \epsilon$$

| Maturity/Tenor | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 24,9% | 24,8% | 24,8% |
| 2 | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,1% | 25,1% | 25,1% | 25,1% | 25,1% | 25,1% | 25,0% | 25,0% |
| 3 | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 24,9% | 24,9% | 24,7% | 24,6% |
| 4 | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 25,0% | 24,8% | 24,6% | 24,4% | 24,2% |
| 5 | 25,0% | 25,0% | 25,0% | 25,0% | 24,9% | 24,9% | 24,9% | 24,9% | 24,9% | 24,9% | 24,8% | 24,7% | 24,6% | 24,4% |
| 7 | 24,9% | 24,9% | 24,9% | 24,9% | 24,8% | 24,8% | 24,7% | 24,7% | 24,6% | 24,6% | 24,3% | 24,0% | 23,7% | 23,4% |
| 10 | 24,8% | 24,8% | 24,8% | 24,7% | 24,7% | 24,6% | 24,6% | 24,5% | 24,5% | 24,4% | 24,0% | 23,5% | 23,1% | 22,8% |
| 15 | 24,6% | 24,5% | 24,4% | 24,4% | 24,3% | 24,3% | 24,2% | 24,1% | 24,0% | 23,9% | 23,5% | 23,2% | 22,7% | 22,4% |
| 20 | 24,3% | 24,3% | 24,1% | 24,1% | 24,1% | 24,1% | 24,1% | 24,1% | 24,1% | 24,0% | 24,0% | 23,5% | 23,0% | 22,4% |
| 25 | 23,7% | 23,8% | 23,8% | 23,8% | 23,8% | 23,7% | 23,7% | 23,7% | 23,6% | 23,6% | 23,0% | 22,4% | 21,9% | 21,5% |
| 30 | 23,4% | 23,4% | 23,3% | 23,2% | 23,0% | 22,9% | 22,7% | 22,5% | 22,3% | 22,2% | 21,6% | 21,3% | 21,1% | 20,8% |

*Figure 4 - Relative differences between LMM+ normal swaption volatilities ante/post +25% relative volatility choc – with adjustment on Rebonato Parameters*

The proportional shock factor is relatively evenly distributed throughout the volatility matrix, especially for the first maturities/tenors.

This approach is suitable only for sensitivities with homogenous chocs on all the swaption surface. In fact, the swaption surface between two closing presents different volatility level movements depending on the maturity and the tenor inducing slope and curvature effects. Moreover, the interest rate curve changes conjointly with volatilities at each closing.

Other factors can be introduced to capture the deformation of the swaption volatility surface slope as well as the joint movements of rates and volatilities:

- Slope deformation factor: $\boldsymbol{\beta_{slope}} = \dfrac{\left(\sigma^*_{\alpha,\beta_{long\ term}} / \sigma^*_{\alpha,\beta_{mid\ term}}\right)}{\left(\sigma_{\alpha,\beta_{long\ term}} / \sigma_{\alpha,\beta_{mid\ term}}\right)}$
- Joint adjustment factor: $\Delta^*_{vol} = \Delta_{vol} \times \dfrac{\sum(F^*_k(0)+\delta)}{\sum(F_k(0)+\delta)}$

Noting that the slope factor is applied only to parameters that capture the volatility surface slope. For instance, below a methodology to adjust the CIR variance process parameters using $\Delta_{vol}$ and $\beta_{slope}$:

$$V_0^{\;*} = \Delta_{vol}^{\;2} \times V_0 \quad ; \quad \theta^* = \beta_{slope}^{\;2} . \Delta_{vol}^{\;2} \times \theta \quad ; \quad \epsilon^* = \Delta_{vol} \times \epsilon$$

| Maturity/Tenor | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 25,14% | 25,14% | 25,14% | 25,14% | 25,14% | 25,13% | 25,13% | 25,12% | 25,13% | 25,14% | 25,10% | 25,04% | 24,98% | 24,92% |
| 2 | 25,22% | 25,25% | 25,25% | 25,27% | 25,28% | 25,29% | 25,30% | 25,32% | 25,33% | 25,35% | 25,39% | 25,39% | 25,33% | 25,26% |
| 3 | 25,33% | 25,35% | 25,37% | 25,39% | 25,39% | 25,40% | 25,40% | 25,42% | 25,43% | 25,43% | 25,37% | 25,30% | 25,19% | 25,05% |
| 4 | 25,41% | 25,45% | 25,48% | 25,50% | 25,53% | 25,54% | 25,58% | 25,57% | 25,57% | 25,54% | 25,41% | 25,18% | 24,99% | 24,78% |
| 5 | 25,50% | 25,54% | 25,56% | 25,56% | 25,55% | 25,56% | 25,58% | 25,58% | 25,58% | 25,57% | 25,52% | 25,50% | 25,37% | 25,19% |
| 7 | 25,63% | 25,65% | 25,68% | 25,68% | 25,66% | 25,63% | 25,59% | 25,58% | 25,54% | 25,47% | 25,28% | 25,03% | 24,76% | 24,50% |
| 10 | 25,90% | 25,94% | 25,93% | 25,90% | 25,88% | 25,87% | 25,90% | 25,82% | 25,77% | 25,71% | 25,42% | 24,93% | 24,53% | 24,32% |
| 15 | 26,20% | 26,17% | 26,16% | 26,16% | 26,10% | 26,04% | 26,03% | 25,99% | 25,89% | 25,80% | 25,49% | 25,14% | 24,74% | 24,44% |
| 20 | 26,51% | 26,48% | 26,42% | 26,45% | 26,44% | 26,41% | 26,47% | 26,49% | 26,50% | 26,52% | 26,65% | 26,17% | 25,61% | 25,04% |
| 25 | 26,44% | 26,52% | 26,55% | 26,63% | 26,68% | 26,74% | 26,82% | 26,86% | 26,82% | 26,76% | 26,29% | 25,66% | 25,11% | 24,65% |
| 30 | 26,80% | 26,79% | 26,73% | 26,64% | 26,53% | 26,37% | 26,21% | 26,02% | 25,88% | 25,74% | 25,25% | 25,00% | 24,96% | 24,78% |

*Figure 5 - Relative differences between LMM+ normal swaption volatilities ante/post +25% relative volatility choc – with adjustment on CIR parameters – and* $\beta_{slope} = 1.1$

Using conjointly the proportional choc factor and the slope factor to adjust LMM+ parameters improve the market consistency. However, the validation of the market consistency of the adjusted parameters also depends on the error levels induced by the reference parameters.
This approach can thus be applied to adjust LMM+ parameters of quarterly closings based on a precise annual calibration

Other LMM+ parameters adjustment methods exist involving partial or total calibration by using fast Gram-Charlier pricing formulas (see Devineau & al, 2017).
It is also possible to construct an LMM+ calibration referential by pre-processing the calibration on multiple financial conditions, then select the nearest neighbour parameters or and fit a machine learning proxy. For that a PCA is necessary to translate each financial condition accordingly to the movements of the yield curve (e.g. 3 principal components) and the swaption volatility surface (e.g. 4 principal components).

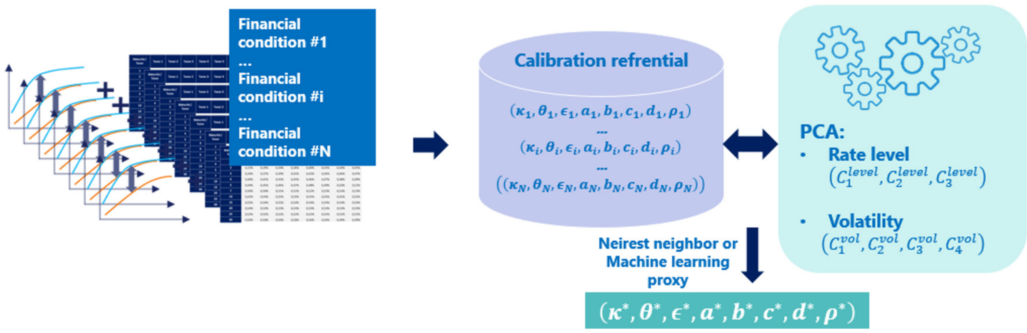



*Figure 6 - Calibration pre-processing methods*

Noting that the parameters adjustment methodologies changes depending on the underlying ESG models used. Indeed, for equity-like models whose parameters are directly linked to market volatilities (TVDV[1] , B&S[2]), the approach is different than for models with stochastic volatilities (LMM+ or SVJD[3]). Models replicating of curves used for inflation and credit spread (resp. 2 factor Vasicek Model and LMN[4]) present a different adjustment methodology for parameters as well.

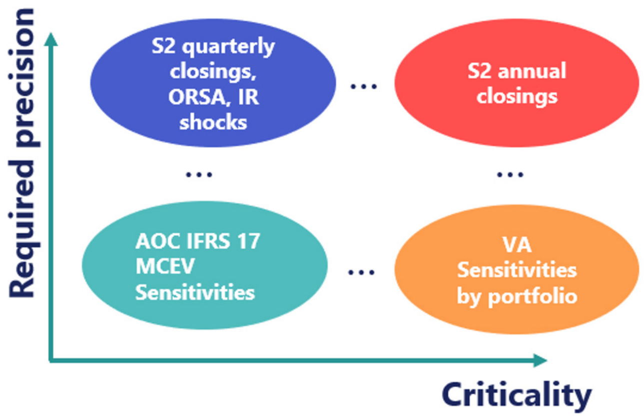
*1 Time Varying Deterministic Volatility (equity model)*
*2 Black and Scholes model (equity/real-estate model)*
*3 Stochastic Volatility Jump Diffusion (equity model)*
*4 Longstaff, Mitall and Neis model*

# 3. CONCLUSION

ESG production in a multi-standard framework becomes operationally intense. Approaches to accelerating ESG production with varying effectiveness have been presented in this article. The industrialisation of ESG production acceleration demand then the construction of a decision tree allowing the operationalisation and governance of the system.

For example, solvency 2 annual closings can be qualified as highly critical and require a high level of precision. Hence, by producing a central economic scenario table without VA, it is possible to use the forward adjustment method to speed up the production of the remaining sensitivities. yet, the parameter adjustment method would be unsuitable.

On the other hand, quarterly solvency 2 closings can be qualified as less critical but still require a high level of precision. In this case, the acceleration of ESG production by the forward adjustment method can be complemented by the use of the parameter adjustment method based on precisely calibrated annual parameters.




Consequently, the industrialisation of the methods presented above in this chapter for accelerating the production of ESG requires :

- Qualifying each sensitivity in terms of criticality, i.e. the degree of precision required according to the issue at stake in the calculation.
- To have pre-assessed the effectiveness of each method in the context of standalone and joint sensitivities.

## Bibliography


- L.Wu, F.Zhang (2006): LIBOR market model with stochastic volatility. Journal of industrial and management optimization 2(2) 199. 1, 2, 3, 4, 5, 6, 7, 8, 14, 15, 16, 20, 24
- K.Mansour (2021) : Etude des modèles de taux G2++ et LMM+ au sein d'un Générateur de Scénarios Economiques risque-neutre et de leurs impacts sur la solvabilité d'un assureur, master thesis.
- L.Devineau, P.E.Arrouy, P.Bonnefoy, A.Boumezoued (2016): Fast calibration of the Libor Market Model with Stochastic Volatility and Displaced Diffusion.

# ANNEX

The LMM+ is an extension of the stochastic volatility Libor Market Model introduced in (whu and Zhang 2006) by adding a displacement factor to generate negative rates.

$$dF_j(t) = \left(F_j(t) + \delta\right) \times \left[\left(\sum_{k=1}^{j}\left(\sum_{q=1}^{2}\xi_j^q(t)\xi_k^q(t)\right)\frac{F_k(t)+\delta}{1+F_k(t)}\right)dt + \sum_{q=1}^{2}\xi_j^q(t)dZ^q(t)\right]$$

Where:

- $\delta$ : Displacement factor (shift) to generate negative rates
- $\xi_j^q(t) := \ g_j(t) \times \sqrt{V(t)} \times \beta_j^q(t)$
- $V(t)$ the stochastic variance following a CIR dynamic

$$dV(t) = \kappa\left(\theta - V(t)\right)dt + \epsilon\sqrt{V(t)}dW_t$$

- $g_j(t)$ the Rebonato volatility function: $g_j(t) = \left(a + bT_{j-m(t)}\right)e^{-cT_{j-m(t)}} + d$
- $\beta_j^q(t)$ vector of inter-forward correlations

In addition, the Brownian motions of the forward rates and the stochastic variance process are correlated: $dW_t . dZ^q(t) = \ \rho dt$.

Hence, the model depends on **8 parameters**[5] $(\boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\epsilon}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{\rho})$ to be calibrated on swaption volatilities at each closing date. The calibration process is based on minimizing the quadratic error between empirical swaption prices and theoretical LMM+ swaption prices.

For that, the theoretical LMM+ price of a European swaption at the instant $t$ maturing at $T_m$ with a tenor $T_n - T_m$ and a strike $K$ is given below:

$$PS_{m,n}(t) = B_s(t)\left[\frac{1}{2}\left(R_{m,n}(t) - K\right) + \frac{1}{\pi}\int_0^\infty \left(\left(R_{m,n}(t) + \delta\right)f_1(u) - (K+\delta)f_2(u)\right) d\,u\right]$$

Where:

- $f_1(u) = \frac{Im\left(e^{-iux}\varphi_X(1+iu)\right)}{u}$ ; $f_2(u) = \frac{Im\left(e^{-iux}\varphi_X(iu)\right)}{u}$
- $\varphi_X$ the characteristic function of the variable $X$ defined by: $X(T_m) = log\frac{R_{m,n}(T_m)+\delta}{R_{m,n}(t)+\delta}$ and depends on the 8 parameters of the LMM+ model.

5. The LMM+ depends on other meta-parameters estimated separately at a specific frequency.

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